Basic Keplerian Orbital Dynamics - Review Sheet

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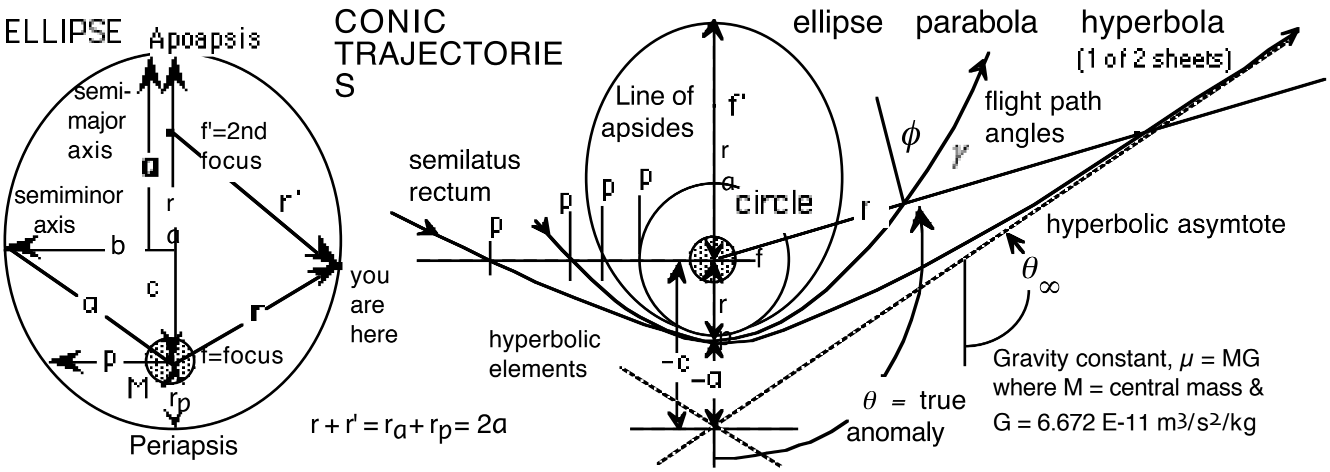


For all Conics: $\mathbf{r} = r_p(1+e)/(1+e\cos\theta) = p/(1+e\cos\theta)$

$\mathbf{r_p} = p/(1+e) = -\mu(1-e)/2E = (1+e)\mu/v_p \quad \cos\theta = (p/r - 1)/e$

$\mathbf{h} = r\,v\cos\phi = r\,v\sin\gamma = r_a v_a = r_p v_p = (p\mu)^{1/2}$

$\mathbf{p} = h^2/\mu = (r_p v_p)^2/\mu = r_p(1+e) = r(1+e\cos\theta) = a(1-e^2)$

$\mathbf{E} = v^2/2 - \mu/r = -\mu/2a = -\mu(1-e)/2r_p \quad \mathbf{a} = -\mu/2E = 1/(2/r - v^2/\mu)$

$\mathbf{e} = (1+2Eh^2/\mu^2)^{1/2} = (p/r_p) - 1 = (1-p/a)^{1/2} = (1-(b/a)^2)^{1/2}$

$\mathbf{v^2} = 2(E+\mu/r) = (2/r - 1/a)\mu \quad \mathbf{v_p}^2 = (1+e)\mu/r_p$

a = semimajor axis b = semiminor axis
c = semi focal separation = $a - r_p = ae$
r_p = periapsis r_a = apoapsis e = eccentricity
h = specific angular momentum
E = Eccentric anomaly M = Mean anomaly
E = specific energy T = period
v = velocity v_e = escape... v_c = circular...
v_∞ = hyperbolic excess vel. n = mean motion

For Circle: $\mathbf{v_c}^2 = \mu/r$ **For Parabola:** $\mathbf{v_e}^2 = 2\mu/r$

For Ellipse: $\mathbf{r_a} = a(1+e) \quad \mathbf{r_p} = a(1-e) \quad \mathbf{e} = (r_a - r_p)/(r_a + r_p)$

$\mathbf{r_p} = r_a(1-e)/(1+e) \quad \mathbf{E} = -\mu(1+e)/2r_a \quad \mathbf{a} = (r_a + r_p)/2$

$\mathbf{v_a}^2 = (1-e)\mu/r_a \quad v_a/v_p = r_p/r_a \quad \mathbf{v_p}^2 = 2(E+\mu/r_p)$

$\mathbf{e} = (r_a - r_p)/2a = (v_p - v_a)/(v_p + v_a) = 1 - v_a^2/(\mu/r_a)$

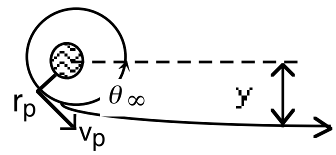
$\mathbf{T} = 2\pi(a^3/\mu)^{1/2} \quad \mathbf{T}_1/T_2 = (a_1/a_2)^{3/2}/(\mu_1/\mu_2)^{1/2} \quad \mathbf{n} = (\mu/a^3)^{1/2}$

$\tan\theta = \tan\phi/(1 - r/p) \quad \tan\phi = e\sin\theta/(1+e\cos\theta)$

CONICS	e	a	E	p
circle	0	r_p	< 0	r_p
elipse	0<1	$> r_p$	< 0	$< 2r_p$
parabola	1	∞	0	$2r_p$
hyperbola	>1	$< -r_p$	> 0	$> 2r_p$

For Hyperbola: $\mathbf{v_\infty} = (v^2 - 2\mu/r)^{1/2} = (v^2 - v_e^2)^{1/2}$

$\cos\theta_\infty = -1/e \quad \mathbf{y} = (a - r_p)\sin(\pi - \theta_\infty) \quad \mathbf{C_3} = v^2 - v_e^2$



Time of Flight Equations: $\mathbf{t_\theta}$ = Time to reach θ from periapsis, **For Circle**: $\mathbf{t_\theta} = (T/2\pi)\theta$

Ellipse: $\tan\boldsymbol{E} = (1-e^2)^{1/2}\sin\theta/(e+\cos\theta) \quad \cos\boldsymbol{\theta} = (e - \cos E)/(e\cos E - 1)$

$\mathbf{t_\theta} = (a^3/\mu)^{1/2}(E - e\sin E) = (T/2\pi)(E - e\sin E) \quad \mathbf{t_\pi} = T/2 = \pi/n = \pi(a^3/\mu)^{1/2}$

$\boldsymbol{M} = 2\pi t_\theta/T = (E - e\sin E)$ [To find $\boldsymbol{E}$ from M, iterate $E_i = M + e\sin E_{i-1}$ until $E_i \approx E_{i-1}$.]

Parabola: $\boldsymbol{D} = p^{1/2}\tan(\theta/2) \quad \mathbf{t_\theta} = (1/\mu)^{1/2}(pD + D^3/3)/2$ D = parabolic eccentric anomaly

$\mathbf{t_\theta} = ((2r_p)^3/\mu)^{1/2}(U + U^3/3)$ where $U = \tan(\theta/2)$

Hyperbola: $\cosh\boldsymbol{F} = \cos E = (e + \cos\theta)/(1 + e\cos\theta)$ F = hyperbolic eccentric anomaly

$\mathbf{t_\theta} = (-a^3/\mu)^{1/2}(e\sinh F - F)$ asymtote: $\cos\boldsymbol{\theta_\infty} = -1/e \quad \cosh F = (1 - r/a)/e$

note: $\sinh(A) = [e^A - e^{-A}]/2$; $\cosh(A) = [e^A + e^{-A}]/2$